Foundations of Operations Research

Master of Science in Computer Engineering

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Thursday 10.15 - 13.15

http://homes.di.unimi.it/~cordone/courses/2014-for/2014-for.html

It is the science of decision-making

It helps people taking better decisions in complex situations, exploiting

- mathematical models
- quantitative methods

to determine the results of each possible decision and
to select the best decision (or at least a satisfactory one)

It is a branch of applied mathematics combining

- economics (to model the decision process)
- knowledge from specific application fields such as industrial engineering, marketing, physics, chemistry, biology, etc. . . (to describe correctly the problems)
- computer science (to solve the problems efficiently)
- business (to turn the suggested decisions into practice)

Decision-making problems

They are characterized by two relevant features

1. the number of solutions:
   - many classical mathematical problems have a single solution:

     Apples cost 1.5 *euro/kg.*
     You buy 3 *kg. of apples*
     How much do you spend?

   - decision-making problems have several feasible solutions and some solutions are better than others:

     Go to the market
     Buy enough food for the whole week
     Spend as little as possible

2. the impact on practical actions:
   - the solution to a classical mathematical problem is
     - an answer to an abstract question
     - an information on what has happened or will happen
   - the optimal solution to a decision-making problem is a guide about how to act in a practical situation

Example: task assignment

- $m = 3$ tasks have to be performed: $T_1, \ldots, T_m$
- $m = 3$ machines are available to perform them: $M_1, \ldots, M_m$
- each task must be assigned to exactly one machine
- each machine can perform exactly one task
- c_{ij} is the cost required to perform task T_i on machine M_j $(i, j = 1, \ldots, m)$

	M_1	M_2	M_3
T_1	2	6	3
T_2	8	4	9
T_3	5	7	8

Decide which machine will perform each task
so as to minimize the total cost

How many solutions are there?

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Exactly $m!$, that is, the number of permutations of the tasks

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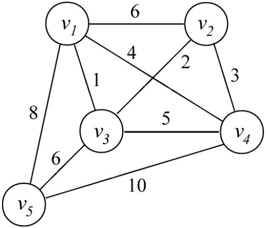
Example: network design

- $n = 5$ towns (nodes) must be linked to each other (directly or not)
- $m = 9$ potential links (edges) exist between towns i and j
- each potential link (i, j) has a cost c_{ij}



Select a subset of edges guaranteeing that each pair of nodes are linked so as to minimize the total cost

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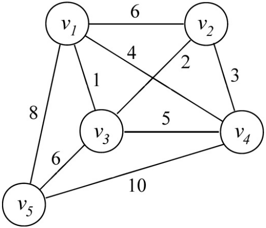

Select a subset of links guaranteeing that each pair of nodes are linked so as to minimize the total cost

How many solutions are there?
At most 2^m, that is the number of subsets of links
(not all subsets guarantee the connectivity)

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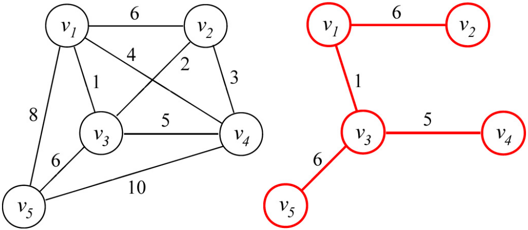

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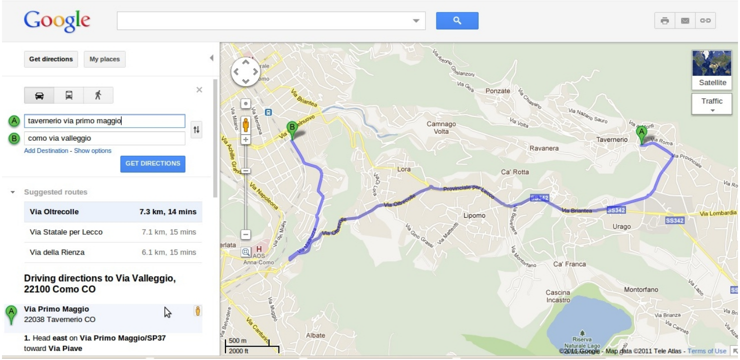
Shortest path

- a digital map of a road network
- an origin location
- a target location

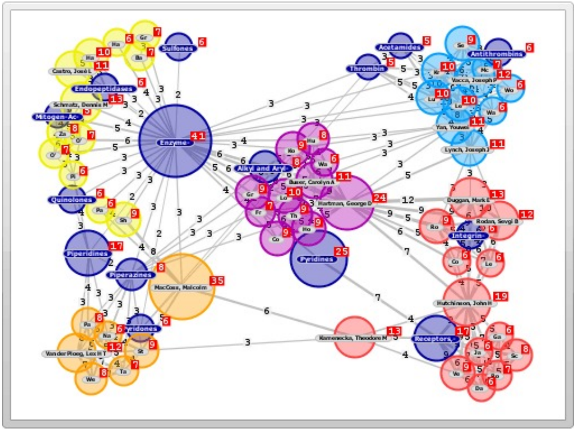


Determine a route from the origin to the target
so as to minimize the travel time (or distance, or cost)

Social networks

A social network

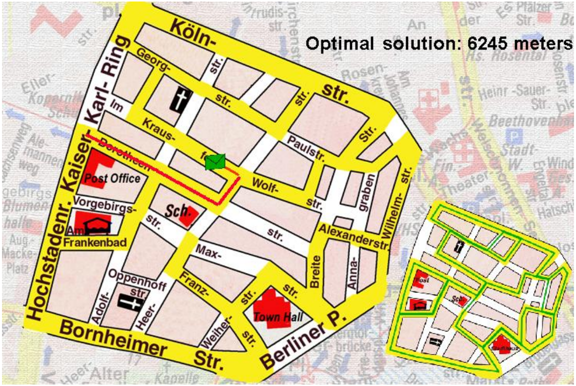
- a set of users
- a set of friendship relations between pairs of users



Find a subset of users who are all pairwise related
so as to maximize their number

Example: mail delivery

- a digital map of a road network
- a starting and arrival point
- a set of streets along which mail must be delivered



Find a close path along all the required streets
so as to minimize its total length

Examples

- Personnel scheduling: determine the weekly shifts for workers covering all daily tasks and respecting labor regulations
  so as to minimize the cost of the personnel

- Service counters:
  determine how many counters to open at each time of the day
  respecting a limit on the average waiting time of the customers
  so as to minimize the cost

- Laptop: choose the components of a laptop
  so as to optimize its total price, weight and performance

A brief historical sketch: the roots

Combinatorial Optimization

Leonhard Euler
(1707-1783)

Linear Programming

Jean Baptiste Joseph Fourier
(1768-1830)

Integer Programming

Diophantus
(200/214-284/298)

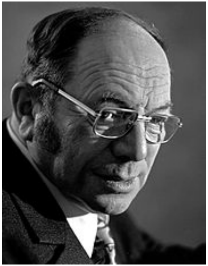
1. 1735: Euler solves the problem of the seven bridges of Königsberg
2. 1826: Fourier solves 3-dimensional Linear Programming problems
3. III century: Diophantus searches for integer solutions to equations

A brief historical sketch: the beginnings

World War II gave the real start to Operations Research, summoning scientists to research on the most efficient conduct of military operations

During the Siege of Leningrad, L.V. Kantorovich (1912-1986) was in charge of the *Road of Life*, an ice road across the frozen Lake Ladoga, which was the only access to the town during winter.

He calculated the optimal distance between cars on ice, depending on ice thickness and air temperature, and personally walked on the ice between the cars to ensure they did not sink.



A brief historical sketch: the developments

Combinatorial Optimization:

- 1926: O. Borůvka finds how to connect all nodes in a network at minimum cost
- 1947: E. Dijkstra finds the shortest path between two places

Linear Programming

- 1939: L.V. Kantorovitch lays the foundations of Linear Programming (Nobel Prize in 1975)
- 1947: G. Dantzig invents the simplex algorithm for linear programs

Integer Programming

- 1958: R. Gomory proposes the cutting plane method
- 1965: E. Balas proposes the branch-and-bound method

A brief historical sketch: business applications

After the war, the substantial increase in the size of companies and organizations gave rise to more complex decision-making problems

OR was widely applied to business, industry, and society

- fast progress in methodologies
- diffusion of computing power (hardware and software)

OR methodologies improve the use of scarse resources
with a significant impact not only for large companies and organizations

Operations Research = Management Science

Rapidly evolving contexts, with high levels of complexity and uncertainty pose a severe challenge to Operations Research

The huge amount of data available (Big Data) with modern information systems opens new avenues (Business Analytics)

A brief historical sketch: business applications

year	company	sector	results
1990	Taco Bell (fast food)	personnel scheduling	7.6M\$ annual savings
1992	American Arlines	design fare structure, overbooking and flights coordination	+ 500M\$
1992	Harris Corp. (semiconductors)	production planning	50% ⇒ 95% orders on time
1995	GM - car rental	use of car park	+50M\$ per year avoided bankruptcy
1996	HP - printers	modify production line	doubled production
1997	Bosques Arauco	harvesting logistics and transport	5M\$ annual savings
1999	IBM	supply chain re-engineering	750M\$ annual savings

Not just money-making

Redesigning Milan's Emergency Service 118