A Computer-based Approach to Teach Tonal Harmony to Young Students

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Abstract: In this paper authors present *Harmonic Touch*, a web platform for the study and practice of tonal harmony. The application is a step-by-step wizard that leads users through 3 experiences towards the discovery of important features of tonal harmony. Leveraging on chord perception, gestural interaction and gamification techniques authors propose an easy and funny approach to a topic otherwise considered too abstract and difficult for young students or amateurs. *Harmonic Touch* has been presented in a workshop to primary and middle school teachers, obtaining a good interest and appreciation as a didactic tool. Feedback received from participants suggests further developments such us expanding the music database and providing teachers with tools to customize *Harmonic Touch* for their teaching needs.

1 INTRODUCTION

Since early childhood, we are continuously and passively exposed to structured music stimuli, and build an inner knowledge about them. The way we learn the music language is a paradigmatic example of implicit learning, i.e. the process of acquiring complex information independently of our awareness of the process of acquisition and of the knowledge acquired. In particular, recent studies have shown that children in preschool age have implicit knowledge of tonality and harmonic structures (Schellenberg et al., 2005; Corrigall and Trainor, 2009).

Also, the child's cognitive development is based on enactive knowledge (Varela et al., 1991): any event is represented through appropriate motor responses, by which children learn to discover the world, the relationships among objects, and cause-effect associations. Active exploration and playful behavior are paramount activities to explore the world and to acquire and master new skills.

Implicit learning and enactive knowledge provide the theoretical and conceptual framework for this work, which presents *Harmonic Touch*, a tool designed for music learning in primary and secondary school. Specifically, the tool is aimed at supporting learning and practicing of tonal harmony.

Building on previous research involving the use of a large scale responsive environment (Mandanici et al., 2016), *Harmonic Touch* introduces three web-based experiences with the use of a touch-screen or computer connected to the internet. The activities are designed for intuitive and immediate interaction, do not rely on previously acquired explicit knowledge of tonal harmony, and exploit the principles of gamification to introduce children to the topic. Moreover, full-body activities mimicking the mouse or touch movements can be arranged for the involvement of a class group and/or for the embodied learning of harmonic concepts (Mandanici et al., 2017).

Harmonic Touch has been preliminary tested in a workshop with primary- and secondary-school teachers. Responses from experimental interviews provided suggestions for improvements and confirmed that this tool be profitably used for introducing tonal harmony features to young students as well as to amateurs and non-musicians.

The remainder of the paper is structured as follows. Section 2 describes the state of the art about the visualization and the understanding of harmonic relationships and progressions. Section 3 provides the theoretical background of the proposed approach for the study of tonal harmony, introducing its perceptual qualities and a spatial representation of the harmonic space. Section 4 describes the architecture of the web-based prototype implementing the mentioned expe-

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riences. Results of the preliminary assessment with teachers are reported in Section 5. Discussion and conclusion follow in Sections 6 and 7, respectively.

2 RELATED WORK

Starting from Euler's *tonnetz* (Euler, 1739), many different theories have been developed for the visualization and the understanding of harmonic relationships and progressions (Cohn, 1998), (Longuet-Higgins et al., 1962), (Schoenberg and Stein, 1969). Similarly, also many computer interfaces and systems aimed at real time generation of harmonic progressions (Eigenfeldt and Pasquier, 2010) or at visualization of harmonies (Chew and Francois, 2005; Hedges and McPherson, 2013) have been realized and tested.

Holland's *Harmony Space* (Holland, 1994) provides a grid of pitches where musical chords can be visualized and selected. It allows the production of scales, chords, inversions and chord progressions. The same interface has been transposed in a large-scale interactive environment where chords can be played through full-body interaction (Holland et al., 2009).

Involving genetic algorithms and Markov models, *Harmonic Navigator* (Johnson et al., 2014) provides both a harmonic palette for the production of follow-up harmonic progressions and a tool for the visualization of the progressions thus generated. Chords are represented through the numbers of harmonic intervals, while colors are used to express the harmonic density. The system employs also motion tracking devices for gestural interaction.

An innovative tool for visualizing the various shifts through harmonic regions in real time is *Mapping Tonal Harmony*[1]. It is based on the visualization of chords placed along fifth circles, with progressions highlighted through popping chord names and brackets. These systems account for the complexity of harmonic relationships in tonal music compositions and, seemingly, are aimed at music students or at people already experienced in tonal harmony features such as chord notation or intervals.

Conversely, *Harmonic Touch* skips harmonic complexity for the benefit of a very simple interface which allows the perception and intuitive positioning of the primary harmonic relationships from which all other more complex chords derive (see Section 3.3). Moreover *Harmonic Touch* is a task-oriented tool, which promotes the knowledge of tonal harmony features through playful experiences focused on concepts such as implicit harmony, harmonic rhythm and melody harmonization.

3 A NEW APPROACH TO THE STUDY OF TONAL HARMONY

Tonal harmony is an idiom, or system of rules, which *"... governs how melodies and chords are organized throughout the duration of a tonal musical composition"* (Butler and Brown, 1994, p.194). Systematically defined in (Rameau, 1722), tonal harmony has been declined through various musical styles, spanning from the Baroque period to contemporary popular songs (Butler and Brown, 1994; Laitz, 2012).

Thus, the tonal system can be considered as a structure of higher level, embracing various stylistic experiences (Brody, 1985). Keys, chords and scales are the abstractions through which tonal harmony is usually described in dozen of treatises and handbooks, e.g. (Rimsky-Korsakov, 1930), (Koechlin, 1930), (Schoenberg, 1983), (Piston, 1948), (Kostka et al., 1984), to name but a few.

But, if on one side these features represent useful tools for analysis and theoretical pedagogy, on the other they miss to explain the perceptual qualities of chord relationships that are peculiar to the tonal system (Butler and Brown, 1994). This formal pedagogical approach has been severely challenged by many didactic scholars (Jaques Dalcroze, 1935) and has been held responsible for failure and disaffection towards harmony studies. Eberlein in particular, in addition to criticizing the abstraction of tonal harmony pedagogy, complains about the use of rules that are in contrast with practice and perception and about the absence of explicit stylistic references in tonal harmony treatises (Eberlein, 1997).

With such premises, it is not surprising that pedagogical approaches to tonal harmony have been scarcely developed outside the strictly professional music curricula, and that little attention has been paid to harmony education programs for children, high school students, and amateurs.

Yet it has been demonstrated that both children and adults who are not musicians have a strong feeling for harmony and are able to recognize the tonic chord (Schellenberg et al., 2005), implicit harmonies (Trainor and Trehub, 1994), and chord progressions (Corrigall and Trainor, 2009).

[1] https://mdecks.com/mapharmony.phtml

3.1 Perceptual Qualities of Tonal Harmony

One of the most outstanding elements of the tonal harmony system is the tonic: *"... the central tone of the key, (it) forms the point of departure from which the other tones move and the goal to which they are directed"* (Aldwell and Schachter, 2003).

Even though the presence of a tonal center is a characteristics common to many musical idioms (modal music, various Eastern musical traditions, a relevant portion of 20th century and contemporary music, etc.), in tonal harmony the tonic chord plays a particularly strong role of closure when preceded by the dominant chord.

To explain the sense of motion delivered to the listeners by tonal music, the scale degrees can be grouped into *stable* ones (I, III, and V) and *active* ones (the remaining degrees of the diatonic scale). Active tones tend to move towards stable ones, thus generating expectation, tension and closure (Aldwell and Schachter, 2003, p.9). Therefore, even if it has been demonstrated that the essence of the strong directed motion typical of the dominant-tonic cadence is linked to the occurrence of "rare" intervals such as the tritone and the semitones (Butler and Brown, 1994, pp.200-201), in a tonal piece there are many other secondary or weaker motions which are learned through experience and that can be recognized when listening (Krumhansl, 2001).

3.2 Theoretical Background

Following H. Riemann's harmonic theory (Riemann, 1896), authors divide the tonal space in primary chords, namely tonic (**T**), subdominant (**SD**) and dominant (**D**), and parallels, i.e. parallel tonic (**Tp**), parallel subdominant (**SDp**) and parallel dominant (**Dp**).

Figure 1 depicts the three tonal functions **T**, **SD**, and **D** (primary chords, namely I, IV, and V degree). The Pythagorean relationships reported below the pitches show the origin of the major harmonic functions and chords starting from *c* (tonic).

Riemann considered the minor chords as the product of the inversion of the harmonic series. Thus, he derived the remaining chords (II, III, and VI degree) from the reversed harmonic series starting from *e* (marked with an asterisk in Figure 2). This is an abstract scheme from which much more complex harmonies can be derived. However, it can fit a number of popular songs as well as classic music harmonization patterns which can be a good starting point for understanding harmonic functions.

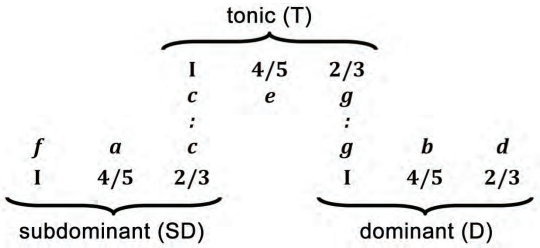

Figure 1: Chart of the relationships of the three tonal functions, also known as primary chords; picture adapted from (Riemann, 1896, p. 7).

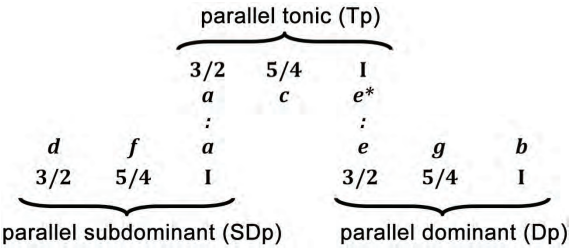

Figure 2: Chart of the relationships of the three parallel chords; picture adapted from (Riemann, 1896, p. 7).

3.3 Representation of the Harmonic Space and Computer Interface

Building on the above presentation, we propose a new approach to the learning of tonal harmony based on musical perception and embodiment of harmonic progressions. If the tonal harmony system is able to deliver different feelings about the various chord progressions, listeners can be trained in recognizing and naming them. This can be done through a spatial arrangement of the musical chords that allows listeners to link the sound of the harmonic progression to a motion scheme. We propose to train this harmonic interaction both through full-body movements and through the touch screen or mouse input.

To this end, the six chords of the harmonic space must be spatially disposed in such a way that each of them can be reached without touching the neighbors. To ease the navigation of the harmonic space and to enhance the perceptual differences between the primary chord zone (all major chords) and the parallel chord zone (all minor chords), we propose to place the chords along a circle with the tonic, dominant and subdominant in the lower part of the circle and the parallels in front of their relatives, as depicted in Figure 3 (white arrows). This is a very simple spatial arrangement with some important peculiarities:

- The user can get acquainted with the sound of the various chords by simply clicking or touching a set of buttons that follow this arrangement: an im-

portant facility for people who cannot play a polyphonic instrument or for children (Manzo, 2014);

- The user can intuitively couple the chord qualities with their location (i.e. the Tonic chord is always in the middle, the other primary chords are one in the left and the other in right, etc.). This can help the memorization of the sound of the various chords as well as the routes of the most important harmonic progressions, as shown by the colored arrows in Figure 3.

4 A WEB-BASED PROTOTYPE

A web prototype has been released in order to show our approach to an audience of music teachers and conduct an early experimentation, as discussed in Section 5. The application was written in HTML5, CSS and JavaScript; server-side interaction, implemented in PHP, was required only to save user performances into a database.

This version was developed as a step-by-step wizard containing self-explanatory descriptions. The prototype focused on three groups of experiences, conceived as a step-by-step process towards harmony awareness:

1. *Recognition of the Implicit Harmony* – The user is asked to match a short music tune with a single chord that, in his/her opinion, best fits the whole melody (Bigand, 1993). The chord is chosen from a set of the 6 primary and parallel chords discussed above. In the interface, chords are represented on the circle of Figure 4, randomly rotated and with no indication of degrees. Understanding the relative layout of chords is left to the user, who can explore them freely during a training phase;

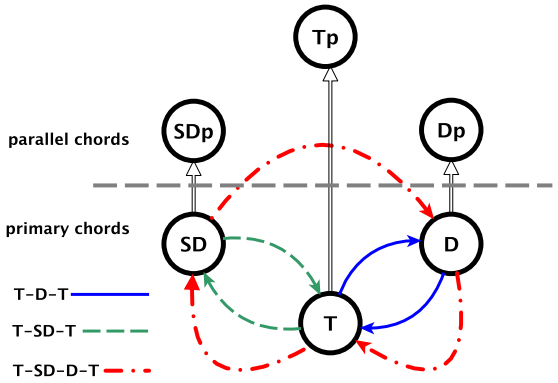

Figure 3: The spatial arrangement of primary and parallel chords with the route of three common chord progressions.

2. *Timed Recognition of Harmonic Changes* – After carefully listening to a complete piece (melody and chords), the user is asked to reconstruct it by moving one step ahead over the map when a new chord is expected. An example of interface is shown in Figure 5. If the click does not occur at the right timing, music stops; if it is performed in advance, a part of the tune is skipped;
3. *Melody Harmonization* – This activity requires to select the right chords at the right timing in order to accompany a known music tune. This last activity is conceived as the natural evolution of the previous ones, focusing on the recognition of the best-fitting chord and the occurrence of harmonic changes. The graphical interface, shown in Figure 6, recalls the chord circle previously mentioned, where the spatial relationship among chords is maintained, but chord functions are made explicit.

In the current implementation, each group of experiences starts with a training phase to make the user get accustomed to the interface and implicitly understand the principles of that activity. After the training phase, three exercises per group are proposed.

The structure of the prototype (e.g., its wizard-like form, the number of exercises, etc.) has been conceived to condense the whole experience within 2 hours, including a brief oral presentation of the project and initial and final surveys. However the educational activities briefly presented here should be expanded into an adequate number of lessons, and music examples should be rethought on the base of the target audience.

Timings and sequences of mouse clicks can be tracked and recorded in a database in order to assess user performances and analyze typical behaviors.

The prototype is publicly available at http://didacta18.lim.di.unimi.it/. At the moment of writing, the interface is available in Italian only, but an English version is expected to be released soon in order to present the approach to the international scientific community.

5 EXPERIMENTATION AT DIDACTA 2018

The mentioned approach was publicly presented and discussed during a workshop titled “Didattica della musica e linguaggi digitali” (music education and digital languages) in occasion of the 2nd edition of Fiera Didacta Italy, Florence, October 18-20, 2018.

Established more than 50 years ago in Germany

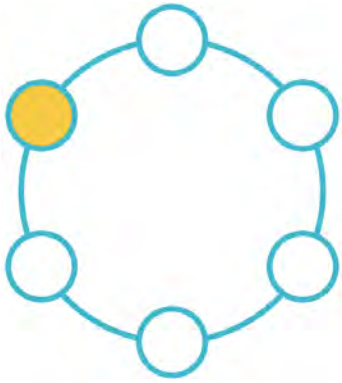

Figure 4: The interface of the first activity group, focusing on the recognition of the implicit harmony of a music tune. No indication is provided to the user about harmonic functions and their spatial disposition, that can be reconstructed only through exploration.

Figure 5: The interface of the second activity group, focusing on the timed recognition of harmonic changes. When a new chord is expected, the user is required to move one step ahead over the map.

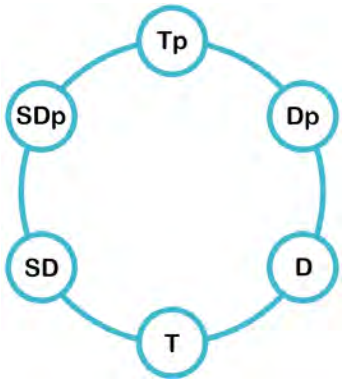

Figure 6: The interface of the third activity group, focusing on the melody harmonization. The same chord circle of Figure 4 is used, but the orientation is kept and harmonic functions are explicit.

and recently exported to Italy, Fiera Didacta[2] is the most important Italian fair focused on education, vocational training and relation among school and work. The goal is to foster innovation and discussion among institutions, teachers, school directors, educators and trainers, as well as professionals and entrepreneurs operating in the school and technology sector.

In this context, the workshop about music education and digital languages aimed to present the prototype described above and to test it with primary- and secondary-school teachers. More in detail, the goals of the workshop were the following:

- To test an early implementation on real users before starting an experimentation phase on young students;
- To profile people interested in this kind of proposal, in terms of age, job, confidence with music practice, etc., thanks to an anonymous questionnaire administered before the experimentation;
- To track their results on a set of increasingly difficult harmony exercises. Please note that the music examples had been conceived for young students, so the performance of domain experts could easily bring to biased results. For this reason, such data will not be commented in this work;
- To gather user observations and suggestions about content and technology-related issues, thanks to a final anonymous survey about the whole experience.

The workshop was attended by an audience of 45 educators, mainly teaching in lower secondary school (57%) and in primary school (25%); only 8% of them came from higher secondary school, 6% from preschool, and 4% from other grades. Concerning gender, 80% in the audience were females, 16% males, and 4% of them did not declare their gender. The mean of participants' age was 49.8 and the median was 51. Concerning the working age, the mean was 22.7 years and the median was 20 years. Educators were mainly music teachers (39 vs. 6), often having experience as music players or singers (35 vs. 10). It is worth noting that, before the experience started, everyone declared to be in favor of using technology in teaching. These data were collected through an initial anonymous survey. The complete dataset of answers is available at http://www.lim.di.unimi.it/data/didacta_survey/.

Figures 7 and 8 show two snapshots of the workshop, with teachers performing exercises of harmony embodiment and practising with the prototype described in Section 4.

[2]http://fieradidacta.indire.it/

Figure 7: Teachers performing an exercise of harmony embodiment during the workshop at Didacta 2018.

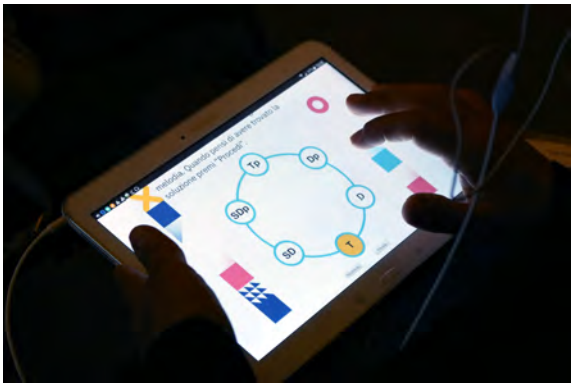

Figure 8: A teacher experiencing the prototype during the workshop at Didacta 2018.

In order to assess the music competences and the opinions about the application of technology in teaching, we administered to participants the same set of questions before and after the proposed activities. Specifically, we asked to what extent technology in education:

Q1. encourages a customized learning;
Q2. provides students with a broad range of information;
Q3. does not encourage the loss of human relationships;
Q4. does not promote distraction in students;
Q5. does not encourage inequality in the possibilities of access to technology;
Q6. prepares students for the use of technology;
Q7. does not encourage an improper use of technological devices;
Q8. increases student involvement;
Q9. supports students with disabilities or special needs;
Q10. does not encourage the violation of privacy.

Please note that in the original survey questions with positive and negative flavor had been mixed. For the sake of clarity, in the list above the questions denoting criticism have been reformulated (e.g., from "promotes distraction" into "does not promote distraction"), and the Likert scale has been turned accordingly.

The results are reported in Fig. 9. From the pre-survey ratings, it can be noted that on average the teachers had in general positive *a priori* expectations about the use of technology for music teaching: positive answers (ratings 1 and 2 on the Likert scale) dominated in all the pre-questions, with the only exception of Q5: this suggests that teachers were particularly concerned with equal opportunities in the access to technology.

Note that the total number of answers to each question varies between the pre- and the post-survey: this is because some of the participants left the workshop without completing the post-survey. Despite the discrepancy, it can be noted that the percentage of positive answers (ratings 1 and 2 on the Likert scale) increased from the pre- to the post-survey, with the only exceptions of questions Q2 and Q6. This indicates that the workshop conveyed in an effective way the potential of the proposed approach. The questions with the highest pre-post positive delta are Q3, Q5, Q7, and Q10. From these results, it can be inferred that the workshop was especially effective in reducing teachers' concerns about possible drawbacks in the use of technology for music teaching. Note that Q5, the only one with a negative balance in the pre-interview, received extremely positive answers in the post-interview.

The prototype raised the interest of the audience, who expressed positive comments through the anonymous final survey. For us, it was particularly interesting to track the opinion of participants on three aspects. First, we asked how clear the overall proposal was, including the aspects of embodiment and the computer-based support activities. Results, expressed in a 5-point Likert scale where 5 is the highest score and 1 is the lowest, are shown in Figure 10. Then, we investigated to what extent the interface was considered user-friendly by users (see Figure 11). Finally, we measured the teaching effectiveness of the solution in users' opinion, obtaining all values greater than or equal to 3, i.e. from neutral to positive responses, as shown in Figure 12.

In the final survey we asked participants also to provide yes/no answers to the following questions:

a. During the introduction, was the focus of the experience clearly explained?
b. Was the content coherent with respect to the expected subject of the experience?
c. Were your expectations satisfied?

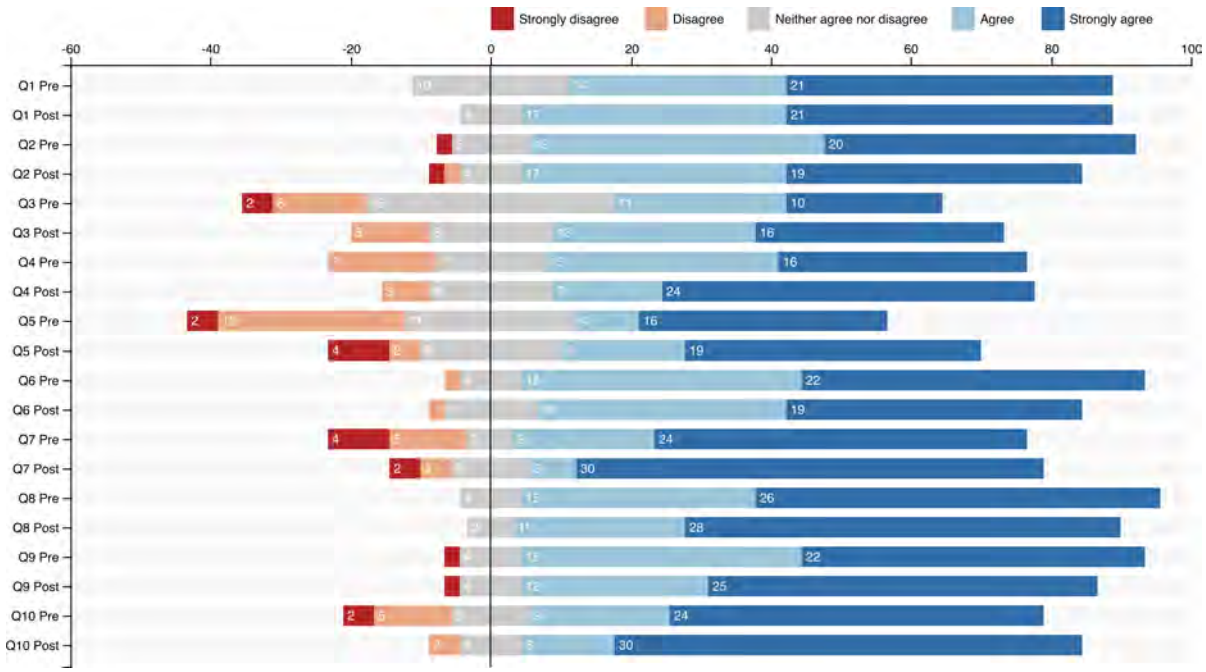

Figure 9: Comparison among the answers provided before and after workshop activities.

d. If you are an educator, are you planning to propose these activities in your classroom?

Most answers were positive: 97.7% for a, 100% for b, 97.7% for c, and 90.7% for d.

6 DISCUSSION

As already mentioned, the primary aim of the preliminary experimentation was to collect feedback from teachers about the effectiveness of the proposed approach. On the other hand, we were not interested in assessing the performance of teachers in completing the activities, mainly because such an analysis would bring to biased results, as teachers are already proficient in the concepts conveyed by the experiences. For this reason, we will not discuss performance results in this work.

The pre- and post-interviews, as well as the informal comments given by teachers at the end of the workshop, provided useful insights for further improvements on the prototype.

Concerning available materials, the solution has to be flexible. The idea is to release a large database of music tunes for each group of activities, trying to meet multiple educational purposes, e.g., considering different students' age, geographical area, culture, and previous music knowledge. Content archive should embrace nursery rhymes, regional tunes, pop songs, well-known music themes, etc.

One desirable additional feature would be a web-based tool that allows teachers to generate their own music materials for the experiences. The tool should include functions and a graphical interface to (i) upload a music tune, and (ii) manually insert a harmonic segmentation the tune, which could then be used in an exercise. As a further alternative to manual segmentation, automatic methods may also be employed (Mauch and Dixon, 2010).

The structure itself of the prototype has to be rethought. First, a wizard-like structure is no more required when a facilitator is available to explain the activities, or users are accustomed to the application features. Needless to say, also the initial and final surveys to profile users and record their suggestions are no more needed.

Any activity should be selectable in an independent way, and without constraints about the number of exercises to administer within a didactic unit. After all, that prototype had been conceived for demonstration purposes, so it had to condense the whole experience in a very limited time span (one of the criticisms emerging from the final survey), whereas harmony awareness is often the result of a long and gradual process.

In order to use *Harmonic Touch* as a teaching tool with children or young students, some reliable metrics have to be developed to automatically assess the performance of the pupil in a given exercise. One possible approach, which is coherent with the gamifica-

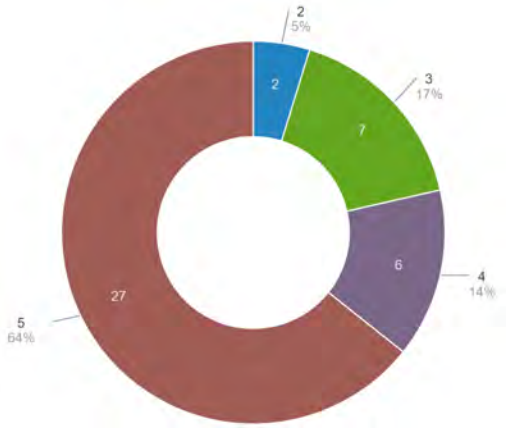

Figure 10: Clarity of the proposal expressed in a 5-point Likert scale.

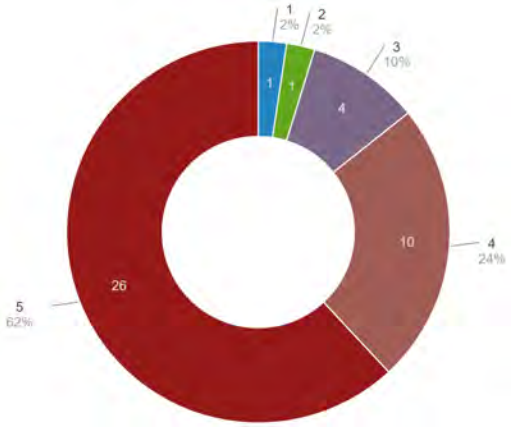

Figure 11: User-friendliness of the proposal expressed in a 5-point Likert scale.

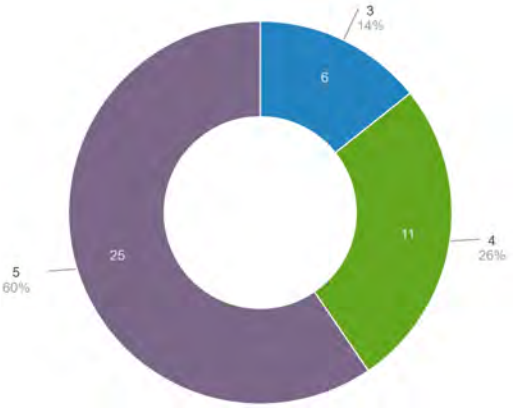

Figure 12: Teaching effectiveness of the proposal expressed in a 5-point Likert scale.

tion approach followed so far, is to generate a score at the end of the exercise. Research on objective assessment of music performance provides examples of how objective descriptors can be correlated to human assessment, e.g., in terms or rhythmic accuracy (Wu et al., 2016).

Also, when the approach is tested on children or young students, the analysis of the outcome of single activities can provide interesting results, that, in turn, can drive further developments or require the implementation of new features.

7 CONCLUSION

In this paper we have presented a prototype of *Harmonic Touch*, a web-based music learning application, focused on tonal harmony. We have discussed the theoretical background which led to the current design of the application, as well as its implementation. We have presented the results of a preliminary evaluation with primary- and secondary-school teachers.

The evaluation showed extremely positive reactions from teachers, with regard to both the teaching potential of the approach and the user friendliness of the current implementation. Furthermore, it provided several insights about possible improvements and further developments.

Further research in the near future will focus on two main directions. On the one hand, the functionalities of *Harmonic Touch* will be expanded and the structure will be redesigned, following the discussion reported above. On the other hand, the application will be tested with children and young students in order to objectively assess its effectiveness in aiding the learning of tonal harmony in the classroom.

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